Loops and Dipoles: A Comparative Analysis

Many factors affect the gain, resistance and general performance of full-wavelength loop antennas. Author Dietrich reports his analysis of loops of various shapes, and compares the characteristics to that of a half-wavelength dipole.

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A long-standing question regarding antennas concerns the precise gain of a one-wavelength loop. An early and oft-quoted figure of approximately 4 dBi was estimated by Lindsay for a one-wavelength circular loop.[1] More recently, Belcher and Casper modeled the full-wavelength loop, using two quarter-wavelength dipoles spaced 0.318 λ—the diameter of a one-wavelength circle.[2] They reported a gain of 4.09 dBi, but later uncovered a computer-programming error. A subsequent calculation, using a dipole spacing of 0.25λ to better approximate a square loop, gave 2.78 dBi of gain.[3] Along these same lines, Lawson calculated a 2.99-dBi gain, using a truncated-cosine current distribution on two quarter-wavelength dipoles to represent the two in-phase sides of the square loop, while omitting the out-of-phase sides.[4] He reasoned that the fields caused by these currents will cancel. Using this result, he further estimated the gain of the circular loop at 3.28 dBi by considering an equivalent length and spacing of the two high-current sides.

Most of the literature on this subject deals with the circular loop, and presents equations for field strength only.[5-7] However, one paper gives a computed curve that shows 3.5 dBi of gain for a one-wavelength circular loop.[8] Another reports a measured value of 3.4 dBi for a 1.1-λ circumference.[9]

So it seems that, depending on the particular model and approximations used, one can arrive at a gain value anywhere from 3 to 4 dBi. Now, ±0.5 dB has little practical significance in most cases, so why all the interest? Well, I think that because this radiator is a basic form and enjoys such widespread use, by itself and in parasitic arrays, we would like to state the characteristics in a precise fashion, just as we can for the half-wavelength dipole antenna. To this end, the following analysis is presented for the gain, radiation resistance and radiation pattern of 1-wavelength loops in all of the popular configurations shown in Fig. 1.

[1]Notes appear on page 26.

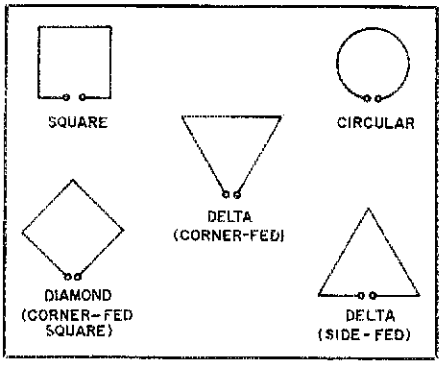

Fig. 1—Five full-wavelength loop shapes are shown here. The Delta loop is an equilateral triangle, with each side being one-third of a wavelength.

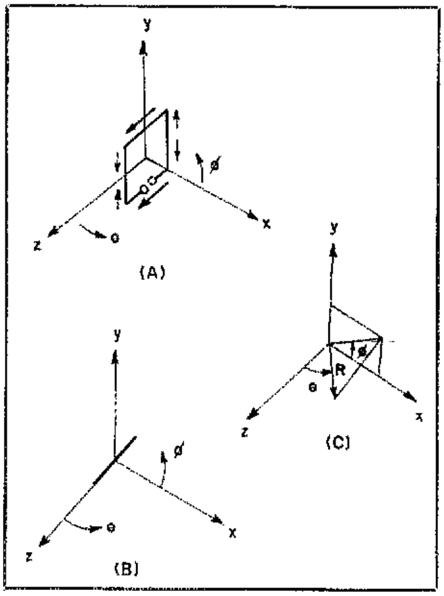

Fig. 2—Geometry used to define antenna position and direction for a square loop (A) and for a half-wavelength dipole (B). At C, the angles θ and φ are defined for any direction.

Formulation of the Problem

The geometry of the problem is shown in Fig. 2A. A square loop is located in the Y-Z plane, with its center at the origin and the feed point at the center of the lower side. A half-wavelength dipole will be referred to, and is oriented as shown in Fig. 2B. In Fig. 2C we can note how to determine the angles θ and ϕ for any direction of R. The angle of R from the Z-axis is expressed as θ. Symbol ϕ is the angle between the X axis and the projection of R onto the X-Y plane.

The currents for the loop are indicated by the arrows on the sides of the square. The heavier arrows indicate the higher current regions. As with the half-wavelength dipole, the amplitude is accurately described by a cosine distribution. This has been verified by measurement on various shapes of loops of total length up to 3 λ.[10] Having established this, the total radiation field was obtained by first finding the field resulting from each side of the loop individually, positioned as in Fig. 2B, then repositioning and reorienting the sides to their appropriate positions of Fig. 2A. Finally, the four resulting fields were added.

Results of the Analysis—The Fields

The fields for the 1-wavelength square loop are given by

$$E_\phi = \cos\phi \sin A \left[\frac{C \cos B - \sin B}{1 - C^2}\right] \quad \text{(Eq. 1)}$$

$$E_\theta = -\cos\theta \sin\phi \sin A \left[\frac{C\cos B - \sin B}{1 - C^2}\right] + \cos B \left[\frac{\cos A - \cos\theta \sin A}{\sin\theta}\right] \quad \text{(Eq. 2)}$$

where

$$A = \frac{\pi}{4}\cos\theta$$

$$B = \frac{\pi}{4}\sin\theta\sin\phi$$

$$C = \sin\theta\sin\phi$$

The total field from these two components is

$$E_t = (E_\phi^2 + E_\theta^2)^{1/2} \quad \text{(Eq. 3)}$$

For comparison, the field for a half-wavelength dipole, as shown in Fig. 2B, is

$$E_\theta = \frac{\cos\left(\frac{\pi}{2}\cos\theta\right)}{\sin\theta} \quad \text{(Eq. 4)}$$

Equations for each antenna have been normalized so the field strength is unity in the maximum direction:

$\theta = 90°$ and $\phi = 0°$

The two similar-looking terms in Eqs. 1 and 2 result from the vertical sides of the loop. The last term in Eq. 2 is caused by the horizontal sides. As has been estimated in the past, the field contribution of the vertical sides is small, having a maximum value of $E_\theta = 0.06$ at $\theta = 36°$ and $\phi = 90°$, that is off the sides at an elevation of $\pm$ 36°. These four low-amplitude lobes slightly cancel the field from the horizontal sides in directions other than the forward direction. This increases the gain a small amount over that when considering the horizontal sides alone.

Radiation Patterns

To calculate the radiation (power) patterns in the principal planes we have the following:

Horizontal ($\phi = 0°$)

$$E_\theta^2 = \left[\frac{\cos\left(\frac{\pi}{4}\cos\theta\right) - \cos\theta\sin\left(\frac{\pi}{4}\cos\theta\right)}{\sin\theta}\right]^2 \quad \text{(Eq. 5)}$$

Vertical ($\theta = 90°$)

$$E_\theta^2 = \cos^2\left(\frac{\pi}{4}\sin\phi\right) \quad \text{(Eq. 6)}$$

These are considerably simpler than Eqs. 1 and 2, because the vertical sides make no contribution in these planes. The polar patterns are shown in Fig. 3. Fields and patterns of other shapes of loops are not given, since they are similar to the square loop. For example, comparing the square and circular loops, we find that the difference in the horizontal patterns is less than 0.2 dB. In the vertical patterns, the maximum difference is in the vertical direction, where the square loop is 3.01 dB down, while the circular loop is 3.74 dB down—a difference of 0.73 dB.

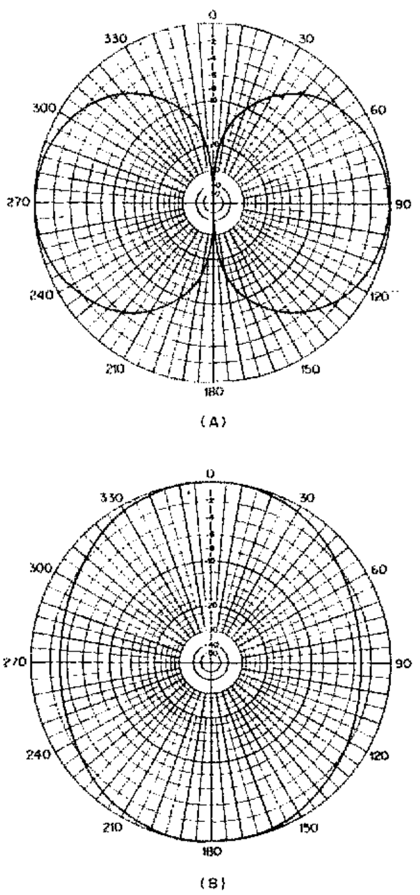

Fig. 3—At A, horizontal radiation pattern for the 1-wavelength square loop. With reference to Fig. 2A, $\phi = 0°$ and the variable angle is θ. The vertical radiation pattern for the 1-wavelength square loop is shown at B. With reference to Fig. 2A, $\theta = 90°$ and ϕ is the variable angle.

For other shapes of loops, the level relative to on-axis in the vertical direction is given in Table 1. Note that this is not relative to the λ/2 dipole, but rather just the amount the pattern is down from the forward (maximum) direction.

Gain Calculation

Gain (over isotropic) may be defined as the ratio of maximum power density to average power density.[11] For the loops under consideration, this is expressed by

$$\frac{1}{G} = \frac{2}{\pi}\int_0^{\pi/2}\int_0^{\pi/2} (E_\theta^2 + E_\phi^2) \sin\theta \, d\theta \, d\phi \quad \text{(Eq. 7)}$$

E_ϕ and E_θ are the normalized fields, which are given for the square loop in Eqs. 1 and 2. Numerical evaluation of Eq. 7 was performed to an accuracy that ensured gain figures to two decimal places. A summary of these results is given in Table 1 for the various 1-wavelength loops and half-wavelength dipole. It shows that the popular square loop has a gain of about 1 dB over a half-wavelength dipole, and that the other shapes are within about one-third of the square configuration. There's nothing too surprising here, except that the gains are a little lower than those often quoted. This results from early estimates of loop gain being based on the circular case, and being a bit high besides.

As a practical matter, it should be noted that dipoles and loops are usually operated at resonant lengths that are very close to one-half and one wavelength, respectively, so the actual gains will be those given in Table 1. For antennas of other lengths, the gain is different. Short dipoles have 1.76 dB of gain, while a 1-wavelength dipole has 3.82 dB of gain. (For patterns of 1-λ and 1.28-λ dipoles, see *The ARRL Antenna Book*, 1984 Ed., pp. 6-7, 6-8.) Loops exhibit similar gain behavior, as shown in Fig. 4, which contains a plot of

Table 1
Summary of Loop and Dipole Characteristics

			Gain Over Isotropic (dB)	*Gain Over Dipole (dB)*	*Radiation Resistance (ohms)*	*Rel. Level at Vertical (dB)*
—	λ/2	Dipole	2.15	0	73	0
□	1 λ	Square loop	3.14	0.99	117	−3.01
○	1 λ	Circular loop	3.49	1.34	133	−3.74
◇	1 λ	Diamond loop	3.14	0.99	117	−2.70
▽	1 λ	Delta loop	2.82	0.67	106	−2.09

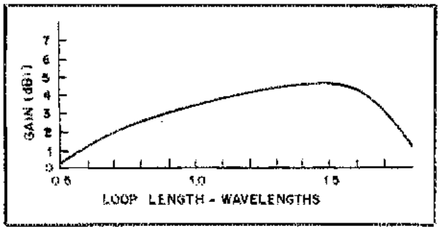

Fig. 4—Computed gain vs. length of a circular loop in the direction broadside to the loop.

computed gain versus length for a circular loop. The drop in gain for short lengths illustrates that loop radiation patterns develop a null in the forward direction as the size is reduced.

Radiation Resistance

Radiation resistance for any antenna may be calculated by:[12]

$$R = \frac{E_0^2 \, r^2}{30 \, G} \qquad \text{(Eq. 8)}$$

where

E_0 is the field strength in the maximum direction, at distance r, with a current of one (1) at the feed point

G is gain over isotropic (not in decibels).

For the square loop,

$$E_0 = 120/\sqrt{2} \quad \text{at } r = 1.$$

Eq. 8 was used to find the radiation resistance for this and other shapes of loops shown in Table 1.

These figures bear the same relation as does the 73 ohms to a dipole; that is, they apply to infinitely thin wire loops, exactly one wavelength long. The input resistance of a practical loop will depend, as with the dipole, on the conductor thickness and length. Thicker conductors and shorter lengths lower the resistance.

Discussion

The results given for all the loops are self-explanatory. However, several qualitative observations can be made. First, all the loops are slightly less directive (broader pattern) in the horizontal plane than is the half-wave dipole, since their horizontal portions are shorter. This is, however, more than compensated for by the "arraying effect" of the two high-current segments (Fig. 2A) that give the vertical pattern directivity. The increased gain is a result of this. It is reflected in the relative vertical level values for each loop (Table 1).

In the case of the square and diamond (corner-fed square), the gain was found to be identical within the accuracy of the calculations. The slightly higher vertical directivity of the square is just compensated by a higher horizontal directivity of the diamond, as a result of its greater horizontal dimensions. Although the gain and radiation resistances are identical for these two, the patterns are not exactly the same, nor do they differ simply as a result of a 45° rotation.

The data shown for the Delta loop applies if the feed point is at the corner or the center of the side. The characteristics are identical, since the current distribution is the same. The Delta loop has the smallest gain because the effective separation of the high-current points is the smallest, as evidenced by the vertical level indicated in Table 1. In contrast, the circular loop has more high-current portion at a greater separation than the others; hence the highest gain.

Additionally, the Delta loop vertical pattern is symmetrical about the horizontal plane, and the direction of maximum radiation is broadside and not offset above or below the horizontal. These two things are not obvious since the triangle shape lacks the vertical symmetry of the other three forms.

Conclusion and Acknowledgment

Values of gain and radiation resistance have been presented for several shapes of 1-wavelength loop antennas. This, and the discussion of their pattern characteristics, gives a clear picture of the behavior. The results should be helpful in determining radiation characteristics of the loop antenna when ground effects must be accounted for, and may also shed some light on the performance of Yagi arrays that use loop elements.

My thanks to reviewer Walter Maxwell, W2DU, for enthusiastically tackling the technical details of the analysis to verify the results and for his interest and suggestions in presenting the material.

Notes

[1]J. E. Lindsay, Jr., "A Parasitic End-Fire Array of Circular Loop Elements," *IEEE Trans. Antennas and Propagation (Communications)*, Vol. AP-15, Sept. 1967, pp. 697-698.
[2]D. K. Belcher and P. W. Casper, "Loops vs. Dipoles—Analysis and Discussion," *QST*, Aug. 1976, pp. 34-37.
[3]J. L. Lawson and P. W. Casper, "Loops vs. Dipoles—Where Will It All End?" *QST*, April 1977, pp. 51-52.
[4]J. L. Lawson, "Yagi Antenna Design: Quads and Quagis," *Ham Radio*, Sept. 1980, pp. 37-45.
[5]E. J. Martin, Jr., "Radiation Fields of Circular Loop Antennas by a Direct Integration Process," *IRE Trans. Antennas and Propagation*, Vol. AP-8, Jan. 1960, pp. 105-107.
[6]J. E. Lindsay, Jr., "A Circular Loop Antenna with Nonuniform Current Distribution," *IRE Trans. Antennas and Propagation*, Vol. AP-8, July 1960, pp. 439-441.
[7]A. Richtscheid, "Calculation of the Radiation Resistance of Loop Antennas with Sinusoidal Current Distribution," *IEEE Trans. Antennas and Propagation (Communications)*, Vol. AP-24, Nov. 1976, pp. 889-891.
[8]S. Ito, N. Inagaki and T. Sekiguchi, "An Investigation of the Array of Circular-Loop Antennas," *IEEE Trans. Antennas and Propagation*, Vol. AP-19, July 1971, pp. 469-476.
[9]J. Appel-Hansen, "The Loop Antenna with Director Arrays of Loops and Rods," *IEEE Trans. Antennas and Propagation (Communications)*, Vol. AP-20, July 1972, pp. 516-517.
[10]P. A. Kennedy, "Loop Antenna Measurements," *IRE Trans. Antennas and Propagation*, Vol. AP-4, Oct. 1956, pp. 610-618.
[11]*The ARRL Antenna Book* (Newington: ARRL, 1984), pp. 2-14, 2-15.
[12]*The ARRL Antenna Book*, pp. 2-5, 2-6.

Jim Dietrich was licensed as WA0RDX in 1967. He is a Life member of the ARRL and holds an Extra Class License. Three of his articles have been published in Ham Radio. *He has also been granted a U.S. patent for a Subharmonic Pumped Mixer Circuit. Jim earned his MSEE at Kansas State University in 1974. He worked for the Boeing Company as part of the Microwave Staff and Electro-Optics Systems organizations until 1981. He has since worked in these areas as a private consultant, and is now program manager at Sigtek Corp., a supplier of commercial and military earth-station equipment.*

Strays

I would like to get in touch with...

☐ anyone involved in an IBM PC user's net. Austin, WB8SXM, and Martha Quinn, KA8LMF, 1806 Vinton, Royal Oak, MI 48067.

☐ anyone who has used a Tano Corp. Dragon computer. M. McDaniel, W6FGE, 940 Temple St., San Diego, CA 92106.

☐ anyone with alignment data for a BC-101 and a Sam SD-10 manual. Lisle T. Hines, K2QLA, 11 Meadow Dr., Homer, NY 13077.

☐ anyone with service information or a circuit diagram for a Central Electronics Model MM-1 multiphase RF analyzer. Jack V. Washburn, W9IVB, 2 Weaver Pl., Urbana, IL 61801.

☐ anyone with an RIT modification for the Heath HW-101 transceiver. Shawn Sabo, KB4KGB, 1555 Mill Run Ct., Lawrenceville, GA 30245.

Next Month in *QST*

Next month, there are plenty of projects to keep your workbench hopping. The amateur looking to add a low-power 40-meter rig to the station will find an article on building a QRP VFO-controlled CW transceiver. Also, there's Part 2 of the principles and building of SSB gear. The beginner and the advanced AMTOR enthusiast will find plenty of helpful hints in "The User's Guide to AMTOR Operation." Or how about learning how to control a CAT with your computer? For the contester, there will be the November Sweepstakes rules and the results of the 1985 International DX Contest. Don't miss 'em!

Feedback: Last month in this space we said that a new column, Under Construction, would debut in the September issue. Because of a scheduling change, it's been delayed a couple of months.